Risk Modeling, Return Forecasting, and Optimal Portfolio Selection: An Empirical Study on Amman Stock Exchange

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Abstract

The main objective of this research is to form an optimal investment portfolio consisting of a number of stocks selected according to specific criteria in Amman Stock Exchange and to test the ability of a various economic models to predict the performance of this portfolio in the foreseeable future. A time series for the return of the selected portfolio and for the return of a market index are formed. A set of tests were conducted to reach a stationary time series return and, then, to follow the Box-Jenkins methodology in order to build predictive models (ARMA) and to examine the residuals of models and to model them using ARCH and GARCH models to reach the best prediction of the performance of the portfolio and the market index in the forecasted periods. The data were tracked on a daily basis for the study sample and the market index simultaneously for a period of three years. Twenty-one companies were selected in the investment portfolio distributed among several sectors. The study concluded that the formed portfolio achieved a good diversification and gave a high return in relation to the lowest possible risk according to the Sharpe scale. Also, it is concluded that the model ARMA (1,1) is the most suitable for estimating market portfolio returns and forecasting risks for the market index return, and ARMA (2,1) and the model ARMA - GARCH are the most capable one of achieving good results that can be relied upon in tracking the performance of the studied investment portfolio.

Key words: Risk Modeling, Forecasting Return, Optimal Portfolio

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نمذجة المخاطر والتنبؤ بالعائد والاختيار الأمثل للمحفظة الاستثمارية: دراسة تطبيقية في بورصة عمان

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مُلخَّص

إن الهدف الرئيس من هذا البحث هو تشكيل محفظة استثمارية مثلى مكونة من عدد من الأسهم يتم اختيارها وفق معايير محددة في بورصة عمان، ومن ثم اختبار قدرة عدد من النماذج الاقتصادية القياسية في التنبؤ بأداء هذه المحفظة على المدى المنظور. تم تشكيل سلسلة زمنية لعائد المحفظة المختارة وسلسلة زمنية أُخرى لعائد مؤشر السوق . تم إجراء مجموعة من الاختبارات للوصول إلى ســـلاسل زمنية مستقرة، ومن ثم اتباع منهجية Box-Jenkins لبنـــاء النماذج التنبؤية (ARMA) وفحص بواقي النماذج ونمذجتها باستخدام نماذج اقتصاد قياســـية (ARCH - GARCH) للوصول إلى أفضل تنبؤ بأداء المحفظة ومؤشر السوق في الفترات اللاحقة. تم تتبع البيانات بشكل يومي لعينة الدراسة ولمؤشر السوق وبشكل متزامن ولمدة ثلاث سنوات. وقد تم اختيار 21 شركة في المحفظة الاستثمارية موزعة بين عدة قطاعات، وتوصلت الدراسة إلى أن المحفظة المشكلة حققت تنوعا جديراً وأعطت عائداً مرتفعاً نسبة إلى أقل خطر ممكن تحقيقة حسب مقياس شارب، وتوصلت الدراسة كذلك إلى أن النموذج (ARMA 1,1) كان الأكثر ملائمة لتقدير عوائد محفظة السوق والتنبؤ بالمخاطر، وكذلك اختيار النموذج و ARMA (2,1) والنموذج (ARMA- GARCH) الأكثر قدرة على تحقيق نتائج جيدة يمكن الاعتماد عليها في تتبع أداء المحفظة الاستثمارية المدروسة.

الكلمات الدالة: نمذجة المخاطر، التنبؤ بالعائد، المحفظة المثلى.

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Introduction:

Forming an investment portfolio is seen as a powerful tool to reduce risks and achieve rewarding returns. Depending on the diversification factor in light of the continuous rise in risks in the business environment, as well as the expansion and diversity of investment opportunities available, can reduce risks to a large extent (Hagin, 2004). Harry Markowitz (1952) emphasized on the concept of diversification in the framework of the investment portfolio theory as it is a key factor necessary to reduce the overall risk of the portfolio. By acquiring investments or financial assets that are not related to each other or have negative returns, the investor can reduce the unsystematic risk to its lowest level (Reilly& Brown, 2012). The concept 'portfolio return' refers to the possibility of experienced profit or loss resulting from the possession of several different investment tools within a portfolio (Amenc & Sourd, 2003). The portfolio return is defined as the weighted average of the returns of the financial instruments included in the formation of the portfolio. The portfolio return is affected by the investor's desires and the extent of their tolerance for the element of risk. Diversification and risk reduction, simultaneously, achieve balanced returns during a certain period of time is a priority. Meanwhile, due to the presence of many differences between investors in terms of priorities related to the investment process, the decision to choose portfolio components is considered one of the most important strategic decisions of the portfolio manager, through which the basic structure is determined portfolio assets, with the aim of maximizing the benefits of diversification and maximizing the expected return of the investment portfolio (Fabozzi & Pachamanova, 2016). Forecasting is a technique that depends on the historical data of a phenomenon as a basis for knowing what the future trends of this phenomenon are expected to be (Maginn, et al., 2007). Forecasting usually relies on ways of making financial and investment decisions to achieve the maximum possible benefit. One of the approaches used to achieve this purpose is to employ the recent models in the time series analysis. The methodology of Box & Jenkins (1976) assumed that autoregressive models and moving averages are appropriate for describing the behavior of financial time series, through their ability to study their fluctuations, depending on a set of steps and procedures that are carried out by adhering to a number of conditions. The most important of these conditions are the stationarity of the time series and the nonexistence of autocorrelation to predict its future trends. Despite the importance of autoregressive models and their contributions, they have been subjected to a number of criticisms (Brockwell, et al., 2009). The most important of them is the assumption of the stationarity of variance, where it is not constant in most cases, especially in the time series representing the returns of securities that are characterized by a high degree of volatility. Consequently, there is a need for models capable of modeling variance to analyze uncertainty. To meet this purpose, Robert Engle (1982) introduced the ARCH model based on the autoregressive representation of conditional variance, and it was later developed by Bollerslev (1986). It became a generalization of the conditional autoregressive model after the variance is constant, by including the expected conditional variance of the previous representations of the residual squares and the previous representations of the variance, which makes the model more comprehensive in practice. Therefore, this study came within the framework of an attempt to determine the most appropriate model that can be used in the process of forecasting the returns of investment portfolios with the aim of framing investment decisions and helping the investor to make rational investment decisions in Amman Stock Exchange, which will have a positive impact on the market as a whole by reducing the cost of transactions and, thus, it increases market efficiency, which in turn is reflected in the local economy. To clarify the previous thoughts, the researcher is going to present the literature related to the concepts of risk and return and their evaluation methods, in addition to the concepts related to the investment portfolio and methods of diversifying it, as well as the definition of time series and the factors affecting them. The study also surveys time series stationarity tests, using the Box & Jenkins methodology, in addition to the most prominent types of Autoregressive Conditional Heteroscedasticity models. Thus, this study came through the formation of an investment portfolio within the conditions of optimizing the return at a specific level of risk (Fabozzi, et al., 2008). The researcher tested the stationarity of time series returns, and investigated it with a set of autoregressive and moving average models, and Autoregressive Conditional Heteroscedasticity Models, with the aim of choosing the most appropriate model to predict the future trends of returns during the studied period.

Research problem:

The motives for investing in securities are varied and numerous, but the achievement of the return is the predominant object. Since this return is linked to the uncertain future, it is therefore subject to fluctuations and carries a degree of risk, and to reduce this risk, investors resort to forming investment portfolios. It is considered an important tool in reducing risks and raising the degree of certainty of the investor. Knowing that this diversification reduces risks, but does not eliminate them, leaves a certain degree of uncertainty related to market conditions that cannot be eliminated by diversification. Thus, the case study of uncertainty needs to use special models that take variance changes into account. In order to make sound investment decisions, the research problem can be clarified by the following main question: is it possible to predict the future trends of investment portfolio returns using the Autoregressive Conditional Heteroscedasticity Models in Amman Stock Exchange? The following sub-research questions are derived from the above question:

1. What is the rank of the model that can be used in modeling and forecasting the returns of investment portfolios?
2. To what extent is the Autoregressive Conditional Heteroscedasticity Models being able to model the returns of investment portfolios and the securities that are included in its formation?
3. What is the extent of compatibility of the actual and predictive values according to the model used for the returns of investment portfolios in Amman Stock Exchange?

Research importance:

The importance of the research is manifested in two aspects. The first one is the scientific aspect, which stems from the fact that it constitutes an extension of a series of research related to econometrics with regard to Amman Stock Exchange, and it deals with the modeling of time series and forecasting its trends, especially in the various aspects of financing and investment operations, in addition to using different methods of mathematical programming employed in the formation of investment portfolios and econometric forecasting models (Ehrhardt & Brigham, 2011). The second is the practical aspect of this study, which stems from the fact that it provides the possibility to verify the effectiveness of the application of prediction models which are based on the principles of autoregressive models and heteroscedasticity in the process of modeling and forecasting the returns of the investment portfolios in this study. This encourages investors to direct their savings towards investment in financial assets through the financial market (Campbell, 2009).

Research objectives:

This study seeks to frame investment decisions related to the formation of investment portfolios according to a reliable methodology based on mathematical methods and the application of econometric models with the aim of maximizing returns and reducing risks, by achieving the following objectives:

1. Verifying the reliability of Autoregressive Conditional Heteroscedasticity Models in predicting the future trends of returns for investment portfolios in Amman Stock Exchange.
2. Determining the rank of the optimal model that can be used in the process of forecasting future returns.
3. Demonstration of the ability of Autoregressive Conditional Heteroscedasticity Models to forecast the returns of investment portfolio that are formulated from a selected sample of equity assets listed in Amman Stock Exchange.
4. Measuring the extent of compatibility between the actual and predictive values of the returns of investment portfolios in Amman Financial Market.

Research hypothesis:

To be able to answer the research questions, the following main hypothesis was formulated:

H0: It is not possible to predict the future trends of investment portfolio returns using Autoregressive Conditional Heteroscedasticity Models in Amman Stock Exchange Market.

The following sub-hypotheses are derived from it:

H01: The time series of portfolio returns do not follow a normal distribution.

Ho2: The time series of portfolio returns is not experiencing a stationary status in second rank.

H03: There is no autocorrelation of the time series return to the random error term for the returns of investment portfolios in Amman Stock Exchange.

H04: There is no effect of the heteroscedasticity of the random error term series for the returns of investment portfolios in Amman Stock exchange.

Review of literature:

Golosnoy, & Gribsch, (2022) proposed direct multiple time series models for predicting high dimensional vectors of observable realized global minimum variance portfolio (GMVP) weights computed based on high-frequency intraday returns. They apply Lasso regression techniques, develop a class of multiple AR(FI)MA models for realized GMVP weights, suggest suitable model restrictions, propose M-type estimators, and derive the statistical properties of these estimators. In the empirical analysis for portfolios of 225 stocks from the S&P 500, they find that their direct models effectively minimize either statistical or economic forecasting losses both in- and out-of-sample as compared to relevant alternative approaches.

Terzi et al., (2021) investigated models and techniques for forecasting volatility in electricity markets and then tested statistical methods based on time series data, with the ARMA-GARCH model being the preferred model, with the goal of identifying optimal methods for this market. During a specific time period, the volatility of the power market and price changes were tested. The authors provide an outline of the research methodologies, having chosen multiple specifications of the ARMA-GARCH model as the most dependable in predicting volatility in the particular market. Forecasting time-varying electricity exchange volatility is critical for all market participants interested in evaluating risk and hedging strategies using variance forecasts.

Bianchi et. al. (2021) explored a portfolio selection issue formulated for irregularly spaced observations. They employed independent component analysis to determine the dependency structure and continuous-time GARCH models to determine the marginal. The researchers studied both market price estimate and simulation in a scenario where the time grid of price quotes varies among assets. As a consequence, they performed an empirical examination of the suggested technique using two high-frequency datasets, which gives superior out-of-sample outcomes than competing portfolio strategies, except in the situation of severe market circumstances with frequent rebalancing.

Yang et al. (2021) investigate the high dimension portfolio optimization by using Markov Switching Copula-based GJR-GARCH model. The proposed model is flexible and can capture the dependence structure that change over time. This model is applied to 8 times series, including DJIA, FTSE, COMEX Gold, US Dollar Index, Crude Oil, and US Bonds (one-month, 2-year, and 5-year). In order to construct a portfolio, first we use GJR-GARCH to capture the volatility of each asset. Then, the Markov Switching copula is used to measure the dependence across assets. Finally, the results from MS-Copula is used to construct portfolios, and Value at Risk and Expected Shortfall are used for optimal portfolio selection.

Zhang & Guo (2018) reviewed several variations or generalizations that significantly improve the performance of Markowitz's mean–variance model, including dynamic portfolio optimization, portfolio optimization with practical factors, robust portfolio optimization, and fuzzy portfolio optimization. Both scholars and practitioners will find the paper valuable in dealing with portfolio selection issues.

An article discussed the weight of a portfolio implanted by Pastpipatkul et al. (2018). The purpose of this article is to assess risks and determine the appropriate weights of a portfolio that includes three instruments: The Thai Stock Exchange, Thai Baht gold, and the 10-year Treasury bond yield. The C-D vine copulas strategy is used in the study to establish the dependence of each pair of instruments, and the Monte Carlo simulation technique is used to produce the simulated data to compute Value at Risk (VaR) and Expected Shortfall (ES). The results demonstrate that there is a weak significant relationship between the Thai Stock Exchange index and Thai Baht gold, as well as a relationship between the 10-year Treasury bond yield and Thai Baht gold. Furthermore, when risk and return are considered, the optimum portfolio allocation is 49.8 percent SET, 18.8 percent Bond, and 31.4 percent Gold.

AL-Najjar (2016) used a similar method in investigating and predicting risks, assumed to be one of the aspects that have a direct function and influence on pricing, risk, and portfolio management, which is volatility modeling in financial markets. As a result, her research will look at the volatility characteristics of Jordan's capital market, such as clustering volatility, leptokurtosis, and the leverage impact. This goal may be achieved by picking symmetric and asymmetric models from the GARCH family of models. This study employs ARCH, GARCH, and EGARCH to explore the behavior of stock return volatility for the Amman Stock Exchange (ASE) from January 1, 2005 to December 31, 2014. The major findings indicate that the symmetric ARCH/GARCH models can capture ASE properties and give better evidence for both volatility clustering and leptokurtic, but EGARCH output shows no support for either.

Adremei & Charles (2014) discussed this issue to predict stock prices using autoregressive models and integrative moving average (ARIMA). This study is implemented based on Nokia stock price data during the time period from (1995) to (2011), in addition to stock price data Zenith Bank for the period from (2006) to (2011). The study concluded, by experimenting with a number of models, that the ARIMA (2,1,0) model is the most appropriate model for forecasting Nokia stock prices with a relatively small margin of error compared to the rest of the models that were used been tested using the ARIMA model (1,1).

Chuang et al. (2012) used the GARCH model to verify the causal relationship between trading volume, stock returns, and volatility of returns, through an applied study on several Asian markets, where well-diversified investment portfolios were created depending on the degree of correlation between its component assets. The study indicated that the average returns among the under-study markets affect the volatility of returns in most markets more than it affects the trading volume, and it affects the trading volume in only two markets: Korea and Thailand. In addition, this study emphasizes the effectiveness of investment portfolios in mitigating the impact of negative yield shocks on both trading volume and yield volatility, but to different degrees.

Al Hamdouni (2011) focused on evaluating the performance of investment portfolios using the risk-adjusted return scale according to the indicators of Sharp, Treynor, and Jensen. The research covered the period of 2009 and extended to two years later. For the purpose of testing the research hypothesis, 116 stock companies were selected as a sample for research whose shares are traded in Amman Stock Exchange. The research aimed to measure the performance of investment portfolios, and it reached the following conclusions: most of the causes of fluctuations in company stock prices resulted from other factors affecting the market, and the low monthly rates of return may have reflected on the market portfolio and appeared with a negative value. Furthermore, the companies did not enhance the realized returns with additional regular returns to cover the decrease of market returns. The research also concluded that using the risk-adjusted return measure in differentiating investment portfolios is better than the use of return and risk separately. The results of the analysis also showed that there was a discrepancy in evaluating the performance of investment portfolios according to Sharp, Treynor and Jensen indicators because each indicator focuses on a specific aspect of risk.

Michael & Manabu (2008) aimed to use volatility that consider many variables, with the aim of using them in managing financial portfolios instead of traditional methods. In this study, the portfolio returns were estimated based on the value-at-risk model, and then these returns were modeled based on the GARCH model. The study was conducted by using financial data of the New York Stock Exchange (NYSE) and the American Stock Exchange (AMEX) during the period from (1998) to (2007). Among the most prominent findings of the study, investment portfolios consisting of a large number of assets are relatively difficult to estimate a model for managing their risks in an efficient manner. Therefore, the use of ARCH/GARCH models and their modifications lead to better results and higher accuracy than if relying only on traditional methods for estimating the returns and risks of a financial investment portfolio. The study also confirmed the possibility of using ARCH\GARCH models in the management process and dynamic adaptation of the investment portfolio assets in order to reduce risks to their minimum limits.

Consequently, this study will aim to update the data that were used in previous studies regarding measuring volatility and forecasting returns for Amman Stock Exchange Index. Also, the period of the study will cover important events that affected the economic conditions regionally and worldwide, such as the COVID-19 pandemic, which still has a significant impact on the performance of companies and the economy in general. Such kind of events is expected to stimulate the importance of estimating and forecasting stock market volatility so that it will ease taking various economic and investment decisions in firms.

Research Methodology:

To answer the research questions and to testits hypotheses, a descriptive analytical approach is followed. Books, theses, and refereed articles were reviewed for the purpose of clarifying theoretical frameworks and key concepts related to research objectives. The sample of this study is composed of shares of companies listed in Amman Stock Exchange and its main index, and then the rates of return are calculated on a daily basis for all stocks, depending on the Microsoft Excel 2016 spreadsheet program. Stock weights were determined in order to build an investment portfolio that achieves the largest possible return at a specific level of risk. In order to study the statistical characteristics and to conduct modeling and forecasting operations, the Eviews 9 program was used, based on the methodology of Box & Jenkins, in the analysis and modeling of time series that take autoregressive and mean regression models ARMA. In addition, the study used conditional autoregressive models for the heteroscedasticity in residual modeling in order to reach statistically significant results that can be interpreted and compared with the reference tables. Furthermore, this study focused on calculating the weight of the stocks involved in the formation of the investment portfolio with the aim of maximizing the portfolio's return and reducing its risk to the minimum level by using non-linear programming procedure (solver function) in the Excel program. Also, the research studied the statistical characteristics of the time series that represent the returns of the investment portfolio of the study sample, plus the stationarity of the time series portfolio’s return, based on the concept of the unit root test.

The research also aims at modeling the portfolio’s series return based on the Box & Jenkins methodology to reach the most appropriate model for describing the volatility of portfolio returns and predicting their future values. It is also worth noting that this research focused on studying the basic properties of the residuals of ARMA models (Brooks, C. 2014) in terms of the extent to which they follow the normal distribution, detecting a state of autocorrelation of errors and testing whether the residuals are subject to a state of heteroscedasticity. Therefore, this study applied the following equations to achieve the objectives of the study:

Stock Returns:

The daily returns for the study sample were calculated based on the following equation:

$$R_t = \frac{(P_{t+1} - P_t)}{P_t}$$

Where,

R_t : daily rate of return per share

P_t : The opening price of the stock during the time period t

P_{t+1}: The closing price of the stock during the time period t

In the same way, the daily returns of the market index portfolio were calculated this way:

Portfolio Returns:

Portfolio return is defined as the profit or loss achieved through an investment portfolio that contains a variety of investments. Therefore, the following equation is used to calculate the Portfolio returns:

$$E(R_p) = \sum_{i=1}^{n} (W_i)\, E(R_i)$$

Where,

$E(R_p)$: Expected Portfolio return

W_t : The weight of stock i in the portfolio.

R_t : daily rate of return per share

Portfolio risk:

The definition of portfolio risk is the possibility of a decline in the value of assets or units of stock held by a company, causing a significant loss for the company in terms of its market value. Portfolio risk is measured by calculating the standard deviation of the portfolio for n securities:

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} W_i W_j \rho_{i,j} \sigma_i \sigma_j$$

Where:

σ_p: Represents the standard deviation of the portfolio

W_i : The weight of stock i in the portfolio

W_j : The weight of stock j in the portfolio

$\rho_{i,j}$: The correlation coefficient between stock i and j

σ_i : The standard deviation of stock i

σ_j : The standard deviation of stock j

In this context, standard deviation alone is not sufficient to calculate the portfolio risk. There is a need to ensure that all the different standard deviations are accounted for by their weights as well as the variance and correlation between the current assets. In this regard, covariance can simply be defined as the extent to which stocks move in the same direction (Cox, D. R., 2011). In other words, it is a measure of how well each of the stocks responds to market trends and other macroeconomic factors.

Sharpe's Model:

Sharpe's model measures the average return earned in excess of the risk-free rate per unit of volatility or total risk: The higher the index, the better:

$$Sharpe = \frac{R_p - R_f}{\sigma_p}$$

Where:

R_p : The Expected return for the portfolio

R_f: Risk-free rate

σ_p : The portfolio standard deviation

Stationarity Models:

A common assumption in many time series techniques is that the data are stationary. A stationary process has the property that the mean, variance, and autocorrelation structure do not change over time. It has several types:

- Autoregressive Models AR(p)

Autoregressive (AR) model is a representation of a type of random process; as such, it is used to describe certain time-varying processes in nature, economics, etc. The autoregressive model specifies that the output variable depends linearly on its own previous values and on a stochastic term (an imperfectly predictable term); thus, the model is in the form of a stochastic difference equation (or recurrence relation which should not be confused with differential equation) (Kirchgassner & Walters, 2007).

$$X_t = c + \sum_{i=1}^{p} \varphi_i X_{t-i} + \varepsilon_t$$

Where

$\varphi_1, \dots \varphi_p$ are the parameters of the model, c is a constant, and ε_t is white noise (Carter and Bruce, 2009).

- **Moving Average Models (q)**

The moving-average model specifies that the output variable depends linearly on the current and various past values of a stochastic (imperfectly predictable) term. The notation MA (q) refers to the moving average model of order q:

$$X_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$

where μ is the mean of the series, the $\theta_1, \dots, \theta_q$ are the parameters of the model, and the $\varepsilon_t, \varepsilon_{t-1}, \dots, \varepsilon_{t-q}$ are white noise error terms. The value of q is called the order of the MA model. Thus, a moving-average model is conceptually a linear regression of the current value of the series against current and previous (observed) white noise error terms or random shocks. The random shocks at each point are assumed to be mutually independent and to come from the same distribution, typically a normal distribution, with location at zero and constant scale (Chatfield, 2000).

- **Mixed Autoregressive Moving Average Models ARMA (p,q)**

Even though the AR(p) and MA(q) models are slightly unrealistic by themselves, this study can mix them to form the extremely useful ARMA(p,q) models. The ARMA(p,q) series $\{X_t\}$ is generated by:

$$X_t = \varphi_t X_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

Where,

$$\begin{cases} \varphi_i : & (i = 1, \dots, p) \\ \boldsymbol{\theta}_j : & (j = 1, \dots, q) \end{cases}$$

represents real numbers, and $\boldsymbol{\varepsilon_t} \sim (\mathbf{0}, \boldsymbol{\sigma_\varepsilon^2})$ (Burke,Orlaith, 2001, p.19).

Dickey – Fuller Augmented Test (ADF)

The ADF test is based on three models to illustrate the time series in this study (X_t):

$$\Delta X_t = \lambda X_{t-1} + \sum_{j=2}^{p} \phi_j \, \Delta X_{t-j+1} + \varepsilon_t$$

$$\Delta X_t = \lambda X_{t-1} + \sum_{j=2}^{p} \phi_j \, \Delta X_{t-j+1} + c + \varepsilon_t$$

$$\Delta X_t = \lambda X_{t-1} + \sum_{j=2}^{p} \phi_j \, \Delta X_{t-j+1} + c + bt + \varepsilon_t$$

The first model contains the intercept, the second one contains the trend and intercept, and the third model is without the intercept and trend. The null hypothesis is tested, which states that there is a unit root, or that the variable is unstable. If the calculated absolute value t is smaller than the absolute value obtained from tables, this requires retesting\, but after taking the differences, and the alternative hypothesis indicates the stationarity of the time series if the calculated t value is greater than the value obtained from tables (Dickey and Fuller, 1981).

Sample selection:

The sample consists of 39 stocks which selected from the companies listed in Amman Stock Exchange, and it was taken into account that the sample is well distributed in all the main and sub-sectors in the market, and it was made sure that the time series for the data of all stocks are compatible with the time period of the study extending from 2/1/2018 until 30/6/2021 on a daily basis. One of the important justifications for choosing the study period is that it includes the period of emergence of the Covid 19 virus. The study examines the extent to which stocks are affected by this epidemic and to identify its reflection on the performance of the market and the national economy as a whole. To test the research hypotheses, companies listed on Amman Stock Exchange were selected as the research population. Several conditions were taken into account for the study sample selection; for instance, the firms that are characterized to be the highest in terms of market capitalization and trading volume, as well as to ensure the availability of data for each company over the period of the research study on a daily basis, starting from 2/1/ 2018 until 30/6/2021. The total number of observations is 835, representing each unit of a working day. The daily opening and closing prices of the companies' shares were mainly used in calculating the daily returns for each share. Also, the parallel daily returns, which represent the performance of the main market index, were calculated.

Table (1) The average return and the standard return for the study sample

No.	Symbol	Average daily Return	Standard Deviation	No.	Symb ol	Average daily Return	Standard Deviation
1	AALU	-0.01%	2.27%	21	JOPI	0.03%	2.56%
2	AIUI	0.05%	1.37%	22	JOPT	0.05%	1.62%
3	AOIC	0.04%	1.40%	23	JPPC	0.22%	1.46%
4	APCT	0.09%	2.21%	24	JTEL	0.01%	1.47%
5	APOT	0.08%	1.72%	25	MEE T	0.06%	1.29%
6	ARBK	0.00%	1.29%	26	NAT A	0.10%	2.10%
7	ARGR	0.06%	1.46%	27	NDA R	0.12%	1.88%
8	ATCO	0.04%	2.12%	28	RUM I	0.03%	1.78%
9	BOJX	-0.04%	1.36%	29	SIJC	0.24%	2.66%
10	CEIG	-0.01%	1.48%	30	SPIC	0.05%	2.56%
11	EXFB	0.09%	1.39%	31	SUR A	0.04%	3.12%
12	IBFM	0.31%	4.20%	32	THB K	-0.06%	1.90%
13	ICMI	0.09%	3.53%	33	THM A	0.00%	2.49%
14	JDFS	-0.09%	2.84%	34	UBSI	0.02%	1.05%
15	JDPC	0.06%	2.11%	35	UINV	0.03%	1.82%
16	JNTH	0.18%	2.28%	36	ULD C	0.00%	2.12%
17	JODA	0.00%	1.30%	37	UNAI	0.25%	2.30%
18	JOEP	-0.04%	1.31%	38	UTO B	-0.10%	2.27%
19	JOIB	0.00%	1.16%	39	VFE D	0.06%	2.15%
20	JOKB	-0.08%	1.86%				

By looking at table (1), which shows the average return and standard deviation of shares, a decrease in the average daily returns can be observed, in addition to the high degree of volatility of these returns measured by the standard deviation, which indicates a high degree of risk.

Investment Portfolio Construction:

Based on the theoretical concepts that were previously presented in this study, the researcher worked on creating an investment portfolio that maximizes the return at a specific level of risk. To achieve this purpose, the mathematical programming method (non-linear) was adopted using Microsoft Excel, with the help of additional functions, specifically the Solver tool to reach the optimal investment ratios in each share. The main objective of forming the optimal portfolio is to maximize the return and to reduce the risk to the lowest possible level. The optimal investment weight that we can invest in each stock can be obtained by setting certain conditions, such as Sharp rate as defining the target function in the Excel program (Solver tool). This works to maximize the return and reduce the risk to the minimum. Additionally, in order not to give negative weights to some stocks, a restriction has been added not to allow for short selling. The results are presented in the table (3).

Table (2) Portfolio components weights

#	Firm Symbol	Weight
1	AALU	0.012%
2	AIUI	6.082%
3	AOIC	1.804%
4	APCT	3.963%
5	APOT	4.156%
6	ARGR	7.362%
7	EXFB	7.177%
8	IBFM	3.951%
9	ICMI	2.662%
10	JDPC	1.498%
11	JNTH	4.339%
12	JOPI	0.175%
13	JOPT	1.769%
14	JPPC	20.564%
15	MEET	7.921%
16	NATA	2.197%
17	NDAR	6.451%
18	RUMI	1.066%
19	SIJC	6.422%
20	UNAI	9.930%
21	VFED	0.500%
	Total	100%

Table (3) Portfolio statistics

Portfolio Return	0.145%
Portfolio Variance	0.003%
Standard Deviation	0.557%
Sharp Ratio	25.307%
Risk-free rate	3.254%
Risk-free rate for daily average	0.004%

It can be seen from table (3) that the risk of the created portfolio is less than the risk of the stocks that composes the portfolio individually, thus, satisfying the condition of reducing the risks to the lowest possible level. Table (2) shows that (18) shares were excluded from the stocks that were not included in the portfolio's composition, by giving them zero weights because they have high risks, low returns, and sometimes negative values, so that the number of shares that make up the portfolio is (21) shares. From table (2), it can be noticed that the lowest risk recorded for the stocks that make up the portfolio amounted to (1.29%), which is (MEET). The weight of investment that has been given for this company is 7.92%, while the higher rate of weight is given to (JPPC), which nearly reached to 20.56% with low level of risk (1.46%), while this study reached the total risk of the portfolio which is (0.557%), as displayed in table (2). Similarly, this study finds that the total return of the portfolio is (0.145%), which is relatively higher than the return of most of the stocks that make up the portfolio. This form of portfolios is in line with the essence of the modern portfolio theory, as Markowitz maintained that choosing the optimal portfolio depends on producing the highest return and the lowest risk.

Statistical Analysis and Time series Modeling:

In this study, several tests of time-series returns were performed, ranging from normality tests to stationarity tests.

- **Normality test of time series returns:**

The normality test was conducted for the data of the study variables represented in the time series of returns of the unweighted index of Amman Stock Exchange (Market portfolio) and the time series returns of the investment portfolio using Jarque-Bera Test. According to this test, when the level of statistical significance is greater than (5%), the null hypothesis is accepted (H0: The research sample does follow a normal distribution), and when the level of significance is less than 5%, then, the null hypothesis is rejected.

- **Normality test of the index returns :**

It is clear from table (4) that the value of the skewness coefficient is (-0.211), which indicates the asymmetry of the distribution of the series around its arithmetic mean. Based on these grounds, the null hypothesis is rejected $(p - value < 5\%)$. The negative skew value tends to the left, meaning that the left tail is longer compared to the normal distribution. The kurtosis coefficient is as follows: $(KU = 6.766, > 3)$. This means that the index series return' distribution is not flat and has a peak (figure 2). As a result, the null hypothesis, which states that the series flatness follows a normal distribution, is rejected. As for the Jarque-Bera test, it is noted that the significance level is less than (5%), so the index returns series does not follow the normal distribution pattern.

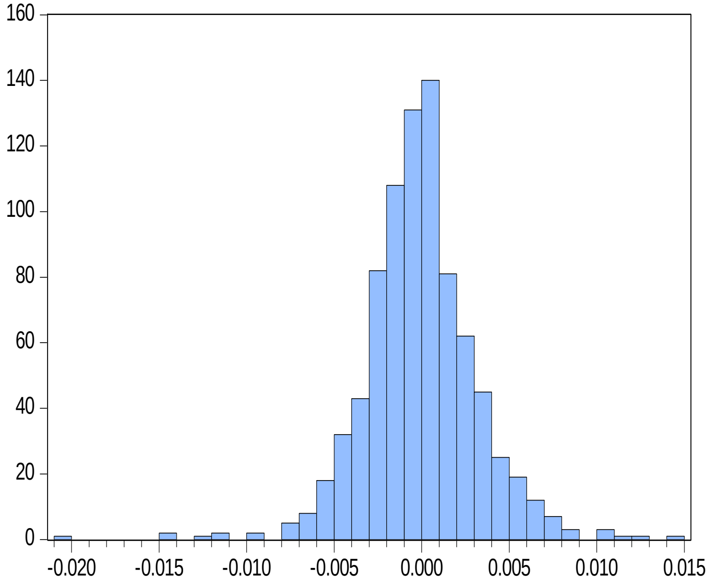


Figure (1) Frequency histogram

Table (4) Descriptive statistics

No. Of Observations	**835**	**Std. Dev.**	**0.00321**
Mean	-0.0000812	Skewness	-0.2110
Median	-0.0000912	Kurtosis	6.76611
Maximum	0.014595	Jarque-Bera	499.669
Minimum	-0.020296	Probability	0.0000

- **Normality test of the investment portfolio's returns :**

It is observed from table (5) that the value of the skewness coefficient reached ($SK = (-0.136977, \neq 0)$, .so the shape of the distribution is asymmetric, and the resulting number is less than zero, which indicates that the residuals of the model are affected by negative shocks more than positive shocks. The kurtosis coefficient reached ($KU = 5.268, > 3$). This indicates that there are outliers in the model results, but these are temporary rather than permanent conditions (figure 2). As for Jarque-Bera's test, it is clear that the level of skewness significance is less than (5%), and, accordingly, the series of returns of the investment portfolio does not follow the normal distribution pattern.

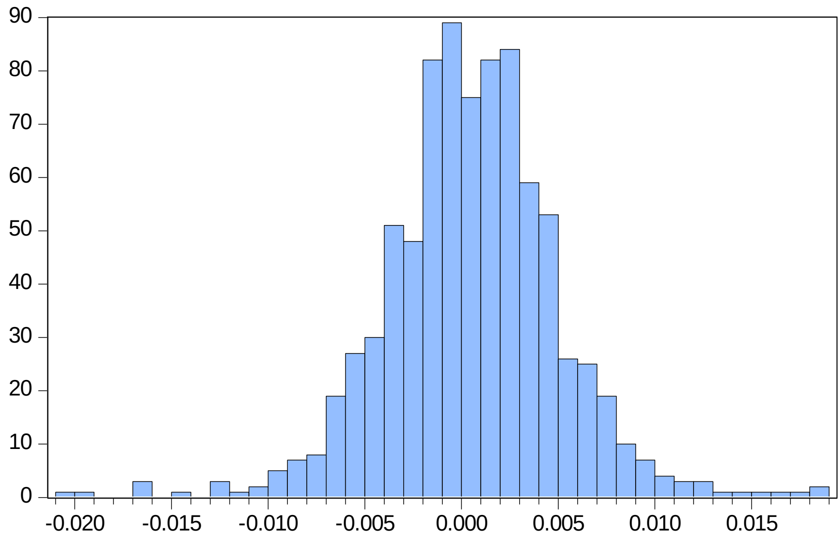


Figure (2) Frequency histogram

Source: EViews 9 outputs

Table (5) Descriptive statistics

No. Of Observations	**835**	**Std. Dev.**	**0.004534**
Mean	0.00050	Skewness	-0.136977
Median	0.00520	Kurtosis	5.268002
Maximum	0.01859	Jarque-Bera	181.5773
Minimum	-0.02082	Probability	0.0000

- **Nonstationary tests of time series:**

The Augmented Dickey-Fuller test is dedicated to studying the stationarity of time series with the aim of detecting the existence of a unit root. This test provides estimation of three models (without intercept and trend, with intercept, and with intercept and trend) using the least squares method at a certain number of differences with time gaps (lags). When the calculated value of Augmented-Dickey-Fuller statistic is smaller than the critical value corresponding to the sample size, the study rejects the null hypothesis (H0: the time series contains a unit root), and the time series is considered stationary in this case. In contrast, the none-stationarity of the time series is recognized when the calculated value is greater than critical value of the Augmented Dickey Fuller statistic test. When conducting the Dickey-Fuller test, it is important to decide which equation to use; the most appropriate equation for the test is the one that includes the intercept term, because the series fluctuates around a non-zero mean. Also, the number of lags has to be determined to be added to the right side of the equation.

- **Nonstationary test for the time series of index returns:**

Time series has stationarity if a shift in the time does not cause a change in the shape of the distribution. Unit roots are one cause for non-stationarity. The researcher will detect the null hypothesis that the time series return has a root test in different models:

- **Unit root test for nonstationary: Market index time series return**

Initially, the maximum lags to be included in the selected model has to be determined, so a test to find the number of lags was performed. The results as follows:

Table (6) :VAR Lag Order Selection Criteria

Lag	AIC	SC	HQ
0	-8.64142	-8.635705	-8.63923
1	-8.73013	-8.718699*	-8.72575
2	-8.73292	-8.715777	-8.72635

AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

From table (6), it is noticed that the maximum lags for this test is (1) according to the Schwarz information criterion, where this figure is determined based on the minimum values of (AIC, SC). Consequently, the null hypothesis could be tested to detect the unit root test according to this criterion.

Table (7) Results of testing the Null Hypothesis of INDEX_RETURN if it has a unit root

ADF	**t-Statistic**	**Prob**	**Augmented Dickey-Fuller test statistic**		
Level			1%	5%	10%
With Intercept	-21.5348	0.00	-3.4380	-2.8648	-2.5686
with Intercept and Trend	-21.9207	0.00	-3.9691	-3.4152	-3.1298
None	-21.5340	0.00	-2.5677	-1.9412	-1.6164

Table (7) indicates the results of the unit root ADF with intercept, with intercept and trend, and without intercept and trend (none). It is clear from the results that the series of daily returns for the main market index is stationary at all levels (1%, 5%, 10%), since the calculated t-value is negative and much less than the ADF test statistics. Hence, the ADF test for the unit root shows that with intercept and Trend is (-21.92), which is less than the critical value (-3.415) at the level of significance (5%); therefore, the time series of returns for the Amman Stock Exchange index is stationary during the study period and does not follow a random walk pattern and the market is incompetent at the weak level. Consequently, the null hypothesis is rejected.

- **Nonstationary test for the time series portfolio' return:**

To determine the number of lags to be included in the model, the following test has to be implemented:

Table (8) VAR Lag Order Selection Criteria

Lag	AIC	SC	HQ
0	-7.949276	-7.943561	-7.947084
1	-8.006522*	-7.995091*	-8.002137*
2	-8.005454	-7.988307	-7.998877

AIC: Akaike information criterion

SC: Schwarz information criterion

HQ: Hannan-Quinn information criterion

It is shown that the number of lags to be included in the model is (1) according to information criterion (AIC, SC).

- **Unit root test of non-stationarity: Portfolio time series return**

It is noted from table (9) that the calculated value of Augmented Dickie-Fuller statistic with (constant and trend) is (-22.865), which is less than the critical value (-3.415) at the level of significance (5%). As a result, the time series of returns for portfolio return is stationary during the study period, and does not follow a random walk pattern. Hence, the null hypothesis is rejected.

Table (9) Null Hypothesis: INDEX_RETURN has a unit root

ADF	t-Statistic	Prob.	Augmented Dickey-Fuller test statistic		
Level			1%	5%	10%
With Constant	-22.6794	0.00	-3.4380	-2.8648	-2.5686
with Constant and Trend	-22.8652	0.00	-3.9691	-3.4152	-3.1298
None	-22.4752	0.00	-2.5677	-1.9412	-1.6164

Autocorrelation and partial autocorrelation significant test:

This test is used to detect the extent to which returns are correlated to each other. Autocorrelation arises between a set of values or observations that are generated based on a random process, during current and previous periods of time, usually referred to as lags periods. This hypothesis is proven in the case the correlation coefficient is statistically different from zero. The null hypothesis (H0: All autocorrelation coefficients are equal to zero) indicates that the returns are independent of each other at a certain number of lags periods. The results of the autocorrelation and partial autocorrelation test usually help to determine the appropriate ARMA ranks to study the time series to be tested and to predict its future trends.

- **Testing the coefficient significance of Autocorrelation and Partial Autocorrelation for the Index time series return.**

In order to study the autocorrelation and partial autocorrelation of portfolio returns, 36 lags periods were defined. The results are exhibited in table (10), which shows that coefficients are significant after the sixth-lag period for both autocorrelation and partial autocorrelation. Therefore, the model, which is proposed to represent index time series, returns to predicting future trends and it is shown to be be valid.

Table (10) Autocorrelation results and partial autocorrelation of Index time series return

Included observations: 835

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.287	0.287	68.958	0.000
		2	0.153	0.077	88.550	0.000
		3	0.115	0.058	99.688	0.000
		4	0.107	0.056	109.31	0.000
		5	0.077	0.022	114.30	0.000
		6	0.039	-0.006	115.61	0.000
		7	0.072	0.049	119.93	0.000
		8	0.096	0.060	127.80	0.000
		9	0.183	0.141	156.05	0.000
		10	0.121	0.024	168.40	0.000
		11	0.063	-0.015	171.81	0.000
		12	0.055	0.001	174.35	0.000
		13	0.088	0.046	180.90	0.000
		14	0.020	-0.041	181.22	0.000
		15	-0.003	-0.022	181.23	0.000
		16	-0.046	-0.069	183.04	0.000
		17	-0.004	-0.002	183.06	0.000
		18	0.024	0.007	183.57	0.000
		19	0.041	0.027	185.02	0.000
		20	0.027	0.004	185.64	0.000
		21	0.019	-0.001	185.96	0.000
		22	0.058	0.033	188.83	0.000
		23	0.010	-0.018	188.92	0.000
		24	-0.025	-0.026	189.45	0.000
		25	-0.012	0.016	189.58	0.000
		26	0.043	0.055	191.20	0.000
		27	0.037	0.016	192.38	0.000
		28	-0.010	-0.038	192.47	0.000
		29	0.016	0.018	192.69	0.000
		30	-0.034	-0.060	193.69	0.000
		31	-0.023	-0.024	194.15	0.000
		32	-0.007	0.004	194.19	0.000
		33	0.007	0.024	194.23	0.000
		34	0.033	0.038	195.19	0.000
		35	0.045	0.022	196.98	0.000
		36	-0.001	-0.036	196.98	0.000

Source: EViews outputs

- **Testing the coefficient significant of Autocorrelation and Partial Autocorrelation for the portfolio time series return**

In order to study the autocorrelation and partial autocorrelation of portfolio returns, 36 lag-periods were identified. The test results are shown in table (11), which shows that coefficients are significant after the five lag-periods for both autocorrelation and partial autocorrelation. Thus, the model, which is proposed to represent portfolio time series, returns to predicting future trends, and it is shown to be valid.

Table (11) Autocorrelation results and partial autocorrelation of portfolio returns

Included observations: 835

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.236	0.236	46.674	0.000
		2	0.089	0.035	53.308	0.000
		3	0.061	0.034	56.388	0.000
		4	0.063	0.041	59.687	0.000
		5	-0.004	-0.034	59.702	0.000
		6	0.015	0.017	59.887	0.000
		7	0.054	0.049	62.378	0.000
		8	0.047	0.024	64.280	0.000
		9	0.076	0.060	69.208	0.000
		10	0.096	0.062	76.994	0.000
		11	0.018	-0.032	77.267	0.000
		12	-0.006	-0.018	77.301	0.000
		13	-0.007	-0.012	77.345	0.000
		14	-0.034	-0.037	78.301	0.000
		15	-0.030	-0.012	79.088	0.000
		16	-0.082	-0.080	84.858	0.000
		17	0.005	0.036	84.876	0.000
		18	0.032	0.032	85.774	0.000
		19	-0.013	-0.033	85.908	0.000
		20	-0.021	-0.012	86.275	0.000
		21	-0.010	-0.002	86.364	0.000
		22	0.012	0.024	86.495	0.000
		23	0.032	0.047	87.381	0.000
		24	-0.008	-0.017	87.430	0.000
		25	-0.012	-0.005	87.546	0.000
		26	0.001	0.013	87.547	0.000
		27	0.045	0.037	89.321	0.000
		28	0.066	0.051	93.113	0.000
		29	0.012	-0.016	93.232	0.000
		30	0.005	-0.010	93.256	0.000
		31	0.050	0.046	95.457	0.000
		32	0.014	-0.022	95.623	0.000
		33	-0.008	-0.012	95.675	0.000
		34	0.026	0.034	96.279	0.000
		35	0.032	0.004	97.158	0.000
		36	-0.014	-0.034	97.333	0.000

Source: EViews outputs

Estimation of ARMA model parameters

After confirming the stability of the time series returns of this study, the Box and Jenkins' methodology (Box and Jenkins, 2008) ought to be employed. At this stage, several tests derived from the group of ARMA models have been conducted to choose the model that associates with the highest value of the log-likelihood and the lowest value for the AIC and BIC information criteria. The following table presents a summary of information for the selected models.

Table (12) Estimating the parameters of the ARMA (p, q) model for modeling the index time series return

Model	Log Likelihood	AIC	BIC	Durbin-Watson stat	R-squared	Parameter significance
AR(1)	3644.904	-8.72312	-8.70614	2.04054	0.082728	Significant*
MA(1)	3638.972	-8.70891	-8.69193	1.939148	0.069575	Significant*
ARMA (1,1)	3648.953	-8.73043	-8.70778	1.962	0.091612	Significant*
ARMA (1,2)	3646.058	-8.72349	-8.70085	2.009955	0.085263	Not significant
ARMA (2,1)	3646.472	-8.72448	-8.70184	2.006958	0.086174	Significant*
ARMA (2,2)	3624.393	-8.67161	-8.64895	1.543704	0.036593	Significant*
ARMA (3,1)	3641.019	-8.71142	-8.68878	1.938374	0.074139	Significant**
ARMA (3,2)	3619.827	-8.66066	-8.63802	1.505111	0.025897	Significant**
ARMA (2,3)	3620.715	-8.66279	-8.64014	1.513045	0.027972	Significant**
ARMA (3,3)	3619.331	-8.65948	-8.63683	1.488507	0.024841	Significant

*Significant level 1%, **Significant level 5%

From the previous table, potential models with significant parameters were identified to be taken the best ones to represent risk modeling and forecast returns. The anticipated model is selected based on the lowest values for each of the information criteria (AIC, BIC), as well as the highest value for the log-likelihood, moreover, from the previous table, by taking the value of Durbin Watson's criterion if it is close to the value of (2), which means that there is no autocorrelation between the random errors of the time series returns. For the information criteria values (AIC, BIC), they indicate to the amount of information that the model loses over time. The lower the values are, the less information the model loses; as a result, the better prediction accuracy is obtained. Consequently, depending on the results of autocorrelation and partial autocorrelation of returns, a comparison was made between the models, which showed that all of their parameters are significant, based on the information criteria (AIC, BIC) as well as on the log-likelihood maximization criterion. The researcher finds that the ARMA (1,1) is the most suitable and most compatible with the criteria that must be considered to give accurate results in modeling risks and forecasting returns of the market index.

It is noted from the previous table that all parameters of the model are statistically significant at the 1% level. It is also clear that the corresponding value of the Durbin-Watson test, which measures the autocorrelation between the estimated values and the previous values of the residuals of the estimated model, is equal to (1.96), which is very close to (2). This means that the residuals of the model do not suffer from autocorrelation to the random error term. Depending on the information criteria SIC and AIC, it is found that the corresponding value for each of them is (-8.7092) and (-8.7375), respectively, and it is significantly low. This means that these criteria indicate the amount of information that the model may lose over time. The less value is obtained, the model is better for prediction. Through the above table, this study finds that the model is suitable for representing the time series of returns of Amman Stock Exchange index during the study period.

Thus, when the selected model ARMA (1,1) is tested, the following results are presented in table (13).

Table (13) Dependent Variable: INDEX_RETURN

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-7.89E-05	0.000179	-0.439843	0.6602
AR(1)	0.656259	0.073880	8.882713	0.0000
MA(1)	-0.416788	0.090576	-4.601518	0.0000
SIGMASQ	9.37E-06	3.46E-07	27.09767	0.0000

Based on these grounds, the proposed model equation is formulated as follows:

$$\begin{aligned} Index\_Return &= -7.89\mathrm{E}-05 \\ &\quad - +0.655259(\mathrm{index\_return})_{t-1} - 0.416788_{\varepsilon-1} \end{aligned}$$

This means that the current value of the market index return is affected by its value from the previous day, in addition to being affected by random values on the previous day as well. Also, a similar analysis of the portfolio time-series returns is performed. The modeling results are presented in the table below:

Table (14) Estimating the parameters of the ARMA (p, q) model for modeling the portfolio time series return

Model	Log Likelihood	AIC	SIC	Durbin-Watson	R-squared	Parameter significancy
AR(1)	3345.437	-8.00584	-7.98885	2.015135	0.055722	Significant*
MA(1)	3343.073	-8.00017	-7.98319	1.966644	0.050348	Significant*
ARMA (1,1)	3346.169	-8.0052	-7.98255	1.992022	0.057382	Not Significant
ARMA (1,2)	3345.687	-8.00404	-7.9814	2.003826	0.056289	Not significant
ARMA (2,1)	3345.785	-8.00428	-7.98163	2.00257	0.056512	Significant*
ARMA (2,2)	3326.712	-7.95859	-7.93594	1.575987	0.012385	Significant*
ARMA (3,1)	3343.523	-7.99886	-7.97621	1.964883	0.051375	Not Significant
ARMA (3,2)	3325.351	-7.95533	-7.93269	1.569546	0.009135	Not Significant

*Significant level 1%

From table (14) it is observed that the optimal model for the representation of the series is ARMA (2,1) as it achieves the highest value of the Log Likelihood and the lowest value of the information standards. Then, after completing the estimation of the model parameters, it is necessary to test the significance of these parameters; the non-significant parameters are deleted and reduced (excluded) or increased (added) in ranks of the model ARMA (p, q). It is noted in table (14) that all models which do not correspond to the characteristics of the optimal model that can be adopted to predict the returns of the investment portfolio have been excluded. Hence, once the optimal model is identified, the model equation can be derived by running the selected model.

Accordingly, when the selected model (ARMA (2,1)) is selected, the following table presents the main results:

Table (15): Dependent Variable: PORTFOLIO RETURN

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000502	0.000205	2.448114	0.0146
AR(2)	0.082114	0.032162	2.553139	0.0109
MA(1)	0.228841	0.030262	7.562009	0.0000
SIGMASQ	1.94E-05	6.91E-07	28.04169	0.0000

Table (15) shows the results of the significant test for the parameters of the model. It is clear that all parameters are significant at the level of statistical significance (0.05). In addition to the fact that the residuals of the estimated model are not auto−correlated, this is confirmed by the value of the Durban Watson test is (2). Also, the values of the information criteria SIC and AIC were significantly lower. From the above presentation, it can be concluded that the appropriate model to represent the time series fluctuations of the returns of the investment portfolio during the study period is mathematically formulated as follows:

$$PORTSER_t = 0.000502 + 0.082114(PORTSER)_{t-1} + 0.228841_{\varepsilon-1}$$

Heteroscedasticity Test for the estimated model residuals

The ARCH test is used based on the Lagrange-Multiplier to find out whether the variance of the random error term is constant over time or not. To perform such a test, the residuals of the estimated model are extracted, and their squares are calculated, and, then, the following regression equation is estimated:

$$\boldsymbol{\sigma_t^2 = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2}$$

The value of (q) is determined depending on the partial autocorrelation function of the variance. Testing the null hypothesis (H0: There is no autocorrelation to the random error term) is rejected when there is at least one parameter of the ARCH coefficient is significant.

- **Heteroscedasticity Test for the estimated model residuals ARMA (1, 1) for the Index time series return**

The researcher wants to test the following hypothesis for the time series index return residuals, where the hypothesis could be stated as follows:

H0: There is no effect for the heteroscedasticity if the significant level of the residuals >5%

H1: There is an effect for the heteroscedasticity if the significant level of the residuals < 5%

Table (16) Heteroscedasticity Test: ARCH

F-statistic	80.58888	Prob. F(1,832)		0.0000
Obs*R-squared	73.64885	Prob. Chi-Square(1)		0.0000
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.55E-06	7.00E-07	9.357990	0.0000
RESID^2(-1)	0.296686	0.033049	8.977131	0.0000
R-squared	0.088308	Mean dependent var		9.33E-06
Adjusted R-squared	0.087212	S.D. dependent var		1.90E-05
S.E. of regression	1.81E-05	Akaike info criterion		-18.99685
Sum squared resid	2.73E-07	Schwarz criterion		-18.98551
Log likelihood	7923.685	Hannan-Quinn criter.		-18.99250
F-statistic	80.58888	Durbin-Watson stat		2.008796

Looking at the probability value in the table (16), it can be noticed that the p-value is less than 5% at the first parameter of the Lagrange-Multiplier test. Accordingly, the null hypothesis is rejected, and the alternative one is accepted, which states that there is an effect of the heteroscedasticity in the residual of the estimated model during the study period.

- **Heteroscedasticity Test for the estimated model residuals ARMA (2,1) for the portffolio time series return**

In the same way, the researcher tested the null hypothesis if the residuals of the model used in estimating the returns of the investment portfolio suffer from the heteroscedasticity. The test results were as follows:

Table (1): Heteroscedasticity Test

F-statistic	9.054629	Prob. F(1,832)		0.0027
Obs*R-squared	8.978680	Prob. Chi-Square(1)		0.0027
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.74E-05	1.48E-06	11.71178	0.0000
RESID^2(-1)	0.103754	0.034480	3.009091	0.0027
R-squared	0.010766	Mean dependent var		0.0000194
Adjusted R-squared	0.009577	S.D. dependent var		0.0000384
S.E. of regression	3.83E-05	Akaike info criterion		-17.50236
Sum squared resid	1.22E-06	Schwarz criterion		-17.49102
Log likelihood	7300.483	Hannan-Quinn criter.		-17.49801
F-statistic	9.054629	Durbin-Watson stat		2.004775
Prob(F-statistic)	0.002699			

Based on the above test where the results are presented, the researcher finds that the value (0.00) is less than 5% at the first parameter of the Lagrange-Multiplier test. Therefore, the researcher rejects the null hypothesis and accept the alternative one, which states that there is an effect of the heteroscedasticity in the residual of the estimated model during the study period. As a result of the previous two tests, it can be noted that it is not possible to rely on ARMA models only for modeling and predicting future returns due to the existence of heteroscedasticity. Correspondingly, in such a case, it is necessary to use other models that can deal with this situation. Consequently, the researcher will use GARCH model for this purpose (symmetric ARCH models).

Estimating the parameters of the GARCH model

The GARCH model can be considered as an alternative one to the ARMA model because it is more comprehensive and it is able to model the fluctuations of stock returns. It is also considered one of the models that helps to deal with the instability of the variance (heteroscedasticity) over the time of the study sample, which belongs to the symmetrical ARCH models.

Table (18): Significance Test results for GARCH (1 ,1) ARMA (1,1) parameters for the time series index return

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000105	0.000226	-0.462727	0.6436
AR(1)	-0.116464	0.057468	-2.026603	0.0427
MA(1)	-0.822248	0.079073	-10.39862	0.0000
	Variance Equation			
C	9.58E-07	3.00E-07	3.195170	0.0014
RESID(-1)^2	0.104236	0.023909	4.359630	0.0000
GARCH(-1)	0.790328	0.049074	16.10480	0.0000
R-squared	0.098812	Mean dependent var		-0.000081
Adjusted R-squared	0.095558	S.D. dependent var		0.003214
S.E. of regression	0.003056	Akaike info criterion		-8.809070
Sum squared resid	0.007762	Schwarz criterion		-8.769438
Log likelihood	3684.787	Hannan-Quinn criter.		-8.793876
Durbin-Watson stat	1.923312			

By looking at table (18), it is noted that all parameters of the equations for the mean and variance are significant at 5% level, and when adding other parameters, the model becomes insignificant. The value of the log likelihood is (3684.78); the information criteria are (-8.809070) and (-8.769438) for each of the AIC and SIC, respectively; and the value of the Durbin-Watson test is (1.92). This leads to confirm that there is no autocorrelation in the residuals of the estimated model; consequently, this research concludes that this model is the most appropriate and compatible with the standards among all models. Through the foregoing and based on comparing different models proposed to represent the index time series returns, it was concluded that the GARCH (1, 1) -ARMA (2,1) model is the appropriate model for representing the time series returns of a market index portfolio for predicting future trends.

Referring to table (18), the mean equation of the ARMA model can be written as follows:

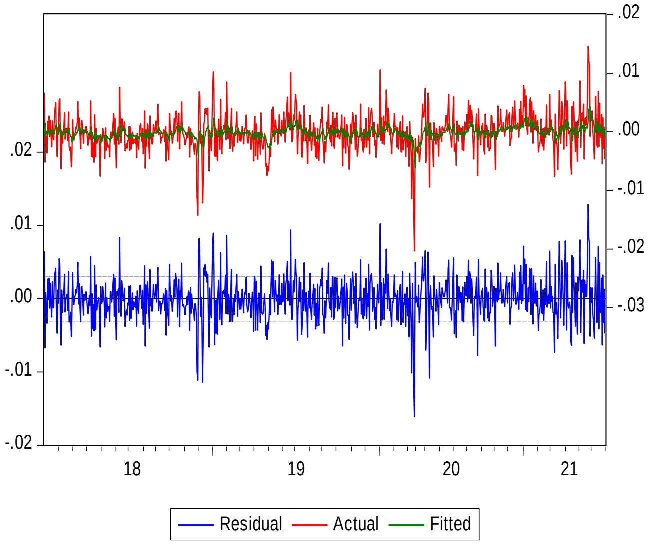
$$(INDEX\_RETURN)_t = -0.000105 - 0.116464(Index\_Return)_{t-1} - 0.822248_{\varepsilon-1}$$

The conditional variance equation can also be formulated as follows:

$$\sigma_t^2 = 0.104236\varepsilon_{t-1}^2 + 0.790328\sigma_{t-1}^2$$

Figure (3) shows the estimated residuals for GARCH (1,1) -ARMA (2,1)) model. It should be noted that the model is remarkably appropriate for capturing the fluctuations' trends of Index time series returns, and this is due to the high volatility that characterizes the time series returns of financial assets.

Figure (3)



The GARCH model was also used to model fluctuations in the portfolio's time-series returns. The results are presented in the following table:

Table (19) Dependent Variable, PORTFOLIO_RETURN

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000662	0.000223	2.963535	0.0030
AR(2)	-0.121996	0.070600	-1.727972	0.0840
MA(1)	-0.805707	0.183269	-4.396317	0.0000
	Variance Equation			
C	1.07E-06	3.73E-07	2.861557	0.0042
RESID(-1)^2	0.055310	0.014291	3.870372	0.0001
GARCH(-1)	0.889760	0.028059	31.71087	0.0000
R-squared	0.058415	Mean dependent var		0.000501
Adjusted R-squared	0.055016	S.D. dependent var		0.004534
S.E. of regression	0.004407	Akaike info criterion		-8.034877
Sum squared resid	0.016141	Schwarz criterion		-7.995246
Log likelihood	3361.561	Hannan-Quinn criter.		-8.019683
Durbin-Watson stat	1.932794			

By looking at the table (19), it is noted that all parameters of the mean and variance equation are significant at the level of statistical significance (0.10), and in the case of adding other parameters, the model becomes insignificant. The log likelihood of possibility is (3361.561), and the value of information criterion are (-8.034877) and (-7.995246) for each of Akaike info. Criterion and Schwarz criterion, respectively. As for the value of the Durban-Watson test, it was high (1.93), and this gives clear evidence that there is no autocorrelation in the residuals of the estimated model, so this model is the closest, accurate, and compatible one with the standards among all previous models. Therefore, it was concluded that Model GARCH (1, 1) – ARMA (2, 1) is the most appropriate and capable of representing the time series returns of the investment portfolio and predicting its future trends. Based on table (19), the mean equation for ARMA (2, 1) can be set as follows:

$$(PORTFOLIO\_RETURN)_t = 0.000662 - 0.121996(portfolio\_Return)_{t-1} - 0.805707_{\varepsilon-1}$$

The conditional variance equation can also be formulated as follows:

$$\sigma_t^2 = 0.055310\varepsilon_{t-1}^2 + 0.889760\sigma_{t-1}^2$$

The figure (4) shows the residuals of the estimated model, and it appears that it is more appropriate in capturing the direction of volatility of the returns of the time series than the breadth of this trend, due to the high volatility that characterizes the returns of the time series of financial assets.

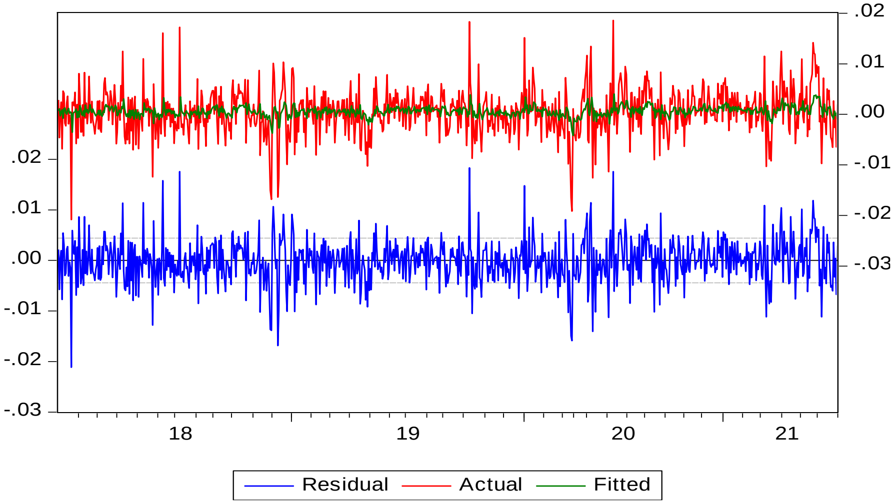


Figure (4) Residuals of the estimated model

Forecasting and comparison of the selected models

Depending on the models that were estimated, this study predicted the returns of each of the market indexes and the investment portfolios during the studied period, compared them with the actual returns, and calculated the percentage of compatibility in the trend.

- **Forecasting the returns of the market index results**

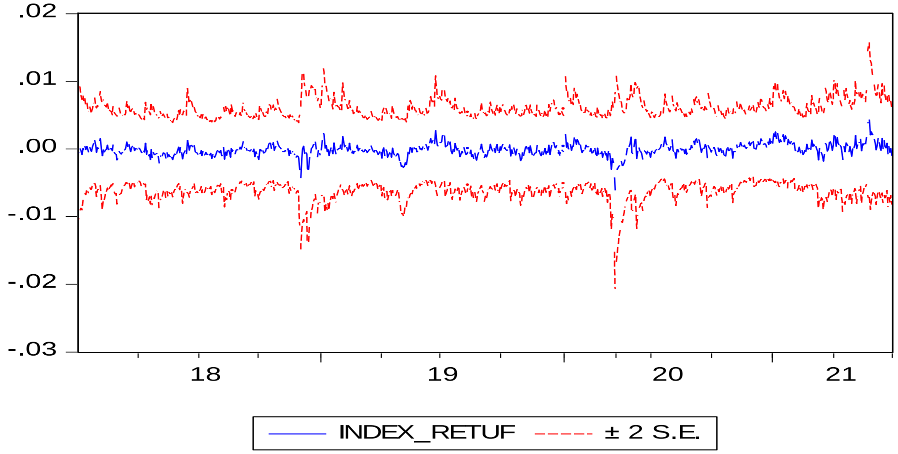
Based on GARCH (1, 1) – ARMA (1,1) Model, and using the E-views 9, in addition to activating the static forecast option, which depends on the actual values of the time series in predicting one future value for each input, the index returns were forecasted during the studied period. The results were presented in the table (20).

Table (20) A comparison in performance between the actual and forecasted returns

	Actual time series return	Forecasted time series return
Average return	-0.00812%	-0.00916%
Standard deviation	0.32117%	0.09498%
Correlation	32%	

Source: outputs of the EVIEWS program and Microsoft EXCEL program.

Figure (5) Forecasting Index time series returns for next periods



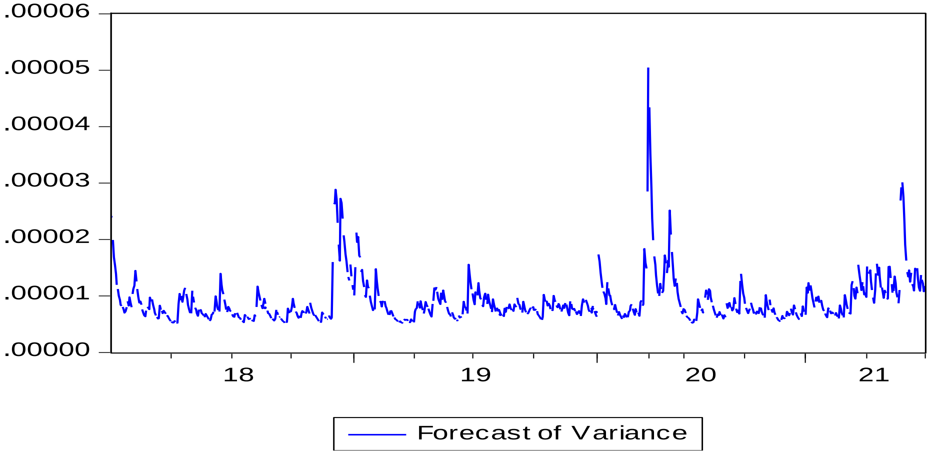


Table (21) Forecasting: Index Return

Actual : Index Return	
Root Mean Squared Error	0.003035
Mean Absolute Error	0.002272
Mean Absolute Percent Error	377.0879
Bias Proportion	0.000008
Variance Proportion	0.550604
Covariance Proportion	0.449389

The upper part of the figure (5) shows the series of predicted returns with two confidence limits and a standard deviation (± 2), and the attached table (21) shows the Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE) as they appear to be low values, which indicates the predictive power of the model, as it reached for each of them (0.003035) and (0.002272), respectively. The Bias Proportion which reached to (0.000008) is also very low, as it appears in the lower section of figure (5). The predicted variance over the length of the studied period, and it is noticeable that there are periods with high volatility and others with low volatility. This is consistent with the results of previous tests.

- **Forecasting portfolio's time series returns results**

Based on GARCH (1, 1) – ARMA (2,1) Model, and using the EViews 9, plus activating the static forecast option, which depends on the actual values of the time series in predicting one future value for each input, the portfolio returns were forecasted during the studied period, and the results are presented in table (22).

Table (22) A comparison in performance between the actual and forecasted returns

	Actual time series return	Forecasted time series return
Average return	0.05005%	0.06062%
Standard return	0.45310%	0.09815%
Correlation	25%	

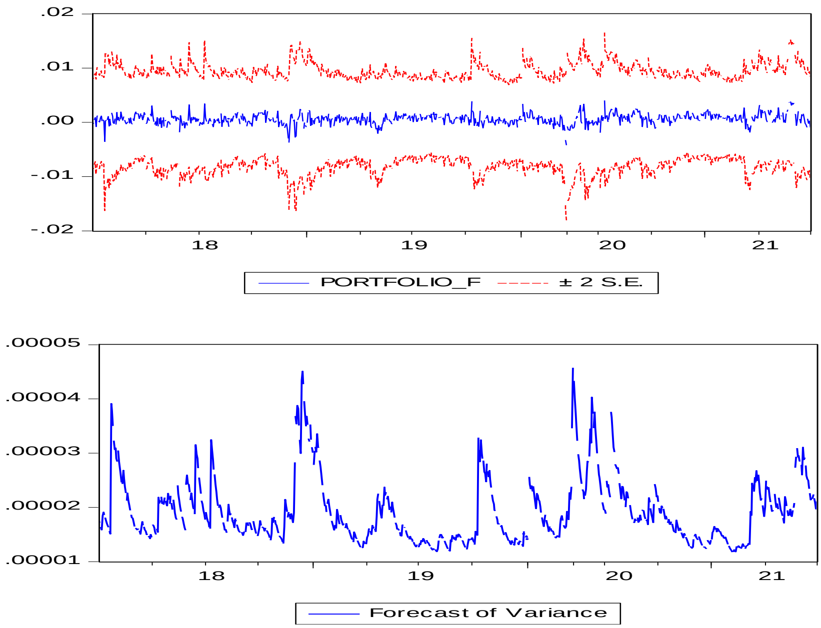
The following table presents the forecasting results for the portfolio time series returns. The upper part of figure (6) displays the series of predicted returns with two confidence limits and a standard deviation (± 2).

Table (23) shows the Root Mean Squared Error (RMSE) and Mean Absolute Error as they appear to be low values, which indicates the predictive power of the model, since each of them reached (0.004397) and (0.003323), respectively, and since Bias Proportion reached to (0.000542), which is also very low. This appears in the lower section of the figure (6), which shows the predicted variance over the length of the studied period. It is noticeable that there are periods with high volatility and others with low volatility, which is consistent with the results of previous tests.

Table (23) Forecast: Index Return

Return	Actual Index
Root Mean Squared Error	0.004397
Mean Absolute Error	0.003323
Mean Absolute Percent Error	207.9153
Bias Proportion	0.000542
Variance Proportion	0.652153
Covariance Proportion	0.347305

Figure (6) Forecasting portfolio time series returns for next periods



Conclusion

This study has achieved the goal of creating a portfolio with a return that is relatively higher than the returns of most of the assets that compose the investment portfolio. The results of the descriptive analysis showed that there is a skewness and a fluctuation in the distribution of the time series return for the market index during the studied period. For the returns of the investment portfolio, it slightly skewed towards the left. The two series do not follow a normal distribution according to the Jarque-Bera test. Furthermore, using the augmented Dickey-Fuller test to detect unit root at three levels (intercept, trend and intercept, none); it was shown that there is no general trend, and the time series of both market index returns and the portfolio are stationary due to the absence of unit root in time series returns, i.e. shocks affecting the series are temporary, and they disappear in the long run, which helps to predict their future trends. In addition, through the results of the unit root test, and the autocorrelation test of the market index returns series, the study concluded that it does not follow the random walk pattern, so Amman stock market is not efficient at the weak form, and stock prices follow a regular movement with a certain pattern. Moreover, the time series returns of the investment portfolio during the studied period are subject to auto−regression from the second rank and moving averages from the first rank. This means that the current value of the portfolio return is affected by its value in the previous two days, in addition to being affected by random variables dating back to the current day and the previous day. Then, the forecasted series by this model (ARMA 2,1) loses two entries.

Furthermore, this study concluded that by testing a set of different models, the time series returns of the market index suffers from first-order auto−regression and first-order moving averages (ARMA 1,1). This indicates that the current value of the index return is affected by its value in the previous one day, in addition to being affected by random variables dating back to the current day and the previous day and the residuals of the estimated model. Correspondingly, fluctuations are represented by the GARCH (1, 1) model. Regarding the time series returns for portfolio, the results of the ARCH- LM test showed that the variance of the random error term is not constant over time. This means that the errors of the ARMA (2,1) model can be represented by the GRCH (1,1) model, which demonstrated an effect of heteroscedasticity. That is, the variance is related to time, which is what characterizes most of the time series that represent the data of financial market indexes. Similarly, for the portfolio time series, the results of the ARCH - LM test showed that the variance is constant, so there is no effect of heteroscedasticity; the ARMA (2, 1) model is considered appropriate to describe the behavior of the portfolio’s return volatility and its future trends. By comparing the results of heteroscedasticity test for the two series, it was concluded that the portfolio has greatly reduced the portfolio's risk compared to the market index's risk. This contributes to achieving a high degree of protection for the investor without reducing the total return of the portfolio.

In the meantime, the estimated models have demonstrated the ability to describe the behavior and volatility of market index return plus portfolio return over the studied period, as well as their ability to provide predictions with relatively small errors, which are indicated by low values of (RMSE) and (MAE). This makes these models capable and effective to determine the future trends of returns accurately. The actual values of returns also remain subject to many random variables, the most notable of which are the weak level of market efficiency, which increase the ability of investors to achieve abnormal returns.

Finally, this study can recommend several things that may be of interest to researchers and those interested in investing in portfolios. For instance, using mathematical programming methods to determine the relative weights of the stocks in the portfolio is essential to maximize the return and reduce the risk to its lowest level. Additionally, the forecasting results obtained by applying the conditional autoregressive models of the none-stationarity of variance, the autoregressive models, and the moving average models must be considered in managing the investment portfolio. Moreover, these tools can be used efficiently and show a satisfactory competency to provide reliable forecasting results in the investment decision-making process. Furthermore, the financial manager of the investment portfolio must reallocate the shares of the portfolio and change their weights whenever necessary to reduce the risks that arise due to price fluctuations in the financial markets.

References:

Al Hamdouni, E. (2011). Evaluating the performance of Investment portfolio / Applied study in Amman Stock Exchange. Journal of Economic and Administrative Sciences, 4(7). Anbar University, Iraq.

AL-Najjar, D. (2016). Modelling and Estimation of Volatility Using ARCH/GARCH Models in Jordan's Stock Market. Asian Journal of Finance & Accounting. 8(1).

Amenc, N., Le Sourd, V. (2003). Portfolio Theory and Performance Analysis, John Wiley & Sons, Chichester, England.

Ayodele A., Aderemi, O. & Charles K. (2014). Stock Price Prediction Using The ARIMA Model, United Kingdom.

Bianchi, F., Mercuri, L. & Rroji, E. (2021) Portfolio Selection with Irregular Time Grids: an example using an ICA-COGARCH (1, 1) approach. Financ Mark Portf Manag . https://doi.org/10.1007/s11408-021-00387-3

Bollerslev. T. (1986). Generalized Autoregressive Conditional Heteroscedasticity, Journal of Econometrica, Volume 31.

Box , G., Jenkins, G. & Reinsel, G.(2008) .Time Series Analysis: Forecasting and Control , Fourth Edition, WILEY INC.

Brockwell, P. & Davis, R. (2009). Time Series: Theory and Methods, second edition, Springer.

Brooks, C. (2014). Introductory Econometrics for Finance, Third edition, Cambridge, Cambridge University Press.

Campbell, J. & Campbell, R. (2009). Analyzing and Managing Risky Investment, Pennsylvania State University.

Carter, M. & Bruce, R. (2009). Op Amps for Everyone. Texas Instruments. pp. 10–11. ISBN 978-0080949482.

Chatfield, Chris (2000). Time series Forecasting, University of Bath, UK.

Chuang, W., Hsiang-His, L, & Rauli, S.. (2012). The bivariate GARCH approach to investigating the relation between stock returns, trading volume, and return volatility. Global Finance Journal 23: 1–15.

Cox, D. (2011). Principals of Applied Statistic, First Edition, Cambridge University Press.

Dickey, D. & Fuller, W. (1981). Likelihood ratio statistics for autoregressive time series with a unit root, Econometrica , 49.

Ehrhardt, M. & Brigham, E. (2011). Financial Management Theory and Practice, 13th Edition, SOUTH-WESTREN Cengage Learning.

Engle, R. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. Econometrica, 50(4), 987–1007. https://doi.org/10.2307/1912773.

Fabozzi, F., Stoyanov, S. & Rachev, T. (2008). Advanced Stochastic Models Risk Management and Portfolio Optimization, John Wiley & Sons.

Fabozzi, F. & Pachamanova, D. (2016). Portfolio Construction and Analytics, John Wiley & Sons, New Jersey, USA.

Hagin, R. (2004). Investment Management: Portfolio Diversification, Risk, and Timing-Fact, John Wiley & Sons, USA.

Markowitz, H. (1952). Portfolio selection. Journal of Finance. 7(1).

Kirchgassner, G. & Walters, J. (2007). Introduction to modern time series analysis, ST Gallen, Berlin , P. 49.

Maginn, J., Tuttle, D., McLeavey, D. & Pinto, J. (2007). Managing Investment Portfolios, 3rd edition, CFA Institute, John Wiley & Sons, New Jersey, USA.

Michael M., & Manabu A., (2008). A Portfolio Index GARCH Model. International Journal of Forecasting. Volume 24.

Pastpipatkul P., Yamaka W. & Sriboonchitta S. (2018) Portfolio Selection with Stock, Gold and Bond in Thailand Under Vine Copulas Functions. In: Anh L., Dong L., Kreinovich V., Thach N. (eds) Econometrics for Financial Applications. ECONVN 2018. Studies in Computational Intelligence, vol 760. Springer, Cham. https://doi.org/10.1007/978-3-319-73150-6_55

Reilly, K. & Brown, C. (2012). Investment Analysis and Portfolio Management, 10th Edition, USA, SOUTH-WESTREN Cengage Learning.

Terzić, I., Jeremić, Z. & Latas, T. (2021). Modelling and Forecasting Volatility on Electric Power Exchange SEEPEX. Management: Journal of Sustainable Business and Management Solutions in Emerging Economies, doi: 10.7595/management.fon.2021.000

Vasyl, G. & Bastian, G. (2022). Modeling and Forecasting Realized Portfolio Weights, Journal of Banking & Finance,106404, ISSN 0378-4266,https://doi.org/10.1016/j.jbankfin.2022.106404.

Yang B., Tarkhamtham P., Phuensan P., & Zhu K. (2021) Portfolios Optimization Under Regime Switching Model: Evidences in the American Bonds and Other Financial Assets. Behavioral Predictive Modeling in Economics. Vol 897. Springer, Cham. https://doi.org/10.1007/978-3-030-49728-6_25.

Zhang, Y., Li, X. & Guo, S. (2018). Portfolio selection problems with Markowitz's mean–variance framework: a review of literature. Fuzzy Optim Decis Making 17, 125–158 (2018).

https://doi.org/10.1007/s10700-017-9266-z

Appendix A: List of the companies that composes the sample study.

#	Symbol	Firm name
1	AALU	ARAB ALUMINIUM INDUSTRY /ARAL
2	AIUI	ARAB UNION INTERNATIONAL INSURANCE
3	AOIC	ARAB ORIENT INSURANCE COMPANY
4	APCT	ARAB COMPANY FOR INVESTMENT PROJECTS
5	APOT	THE ARAB POTASH
6	ARBK	ARAB BANK
7	ARGR	ARAB JORDANIAN INSURANCE GROUP
8	ATCO	INJAZ FOR DEVELOPMENT & PROJECTS
9	BOJX	BANK OF JORDAN
10	CEIG	CENTURY INVESTMENT GROUP
11	EXFB	EXFB OPEN FUND
12	IBFM	INTERNATIONAL BROKERAGE & FINANCIAL MARKETS
13	ICMI	INTERNATIONAL FOR MEDICAL INVESTMENT
14	JDFS	JORDANIAN DUTY FREE SHOPS
15	JDPC	JORDAN DECAPOLIS PROPERTIES
16	JNTH	AL-TAJAMOUAT FOR CATERING AND HOUSING CO PLC
17	JODA	JORDAN DAIRY
18	JOEP	JORDAN ELECTRIC POWER
19	JOIB	JORDAN ISLAMIC BANK
20	JOKB	JORDAN KUWAIT BANK
21	JOPI	THE JORDAN PIPES MANUFACTURING
22	JOPT	JORDAN PETROLEUM REFINERY
23	JPPC	JORDAN POULTRY PROCESSING & MARKETING
24	JTEL	JORDAN TELECOM
25	MEET	METHAQ REAL ESTATE INVESTMENT

26	NATA	NATIONAL ALUMINIUM INDUSTRIAL
27	NDAR	NUTRI DAR
28	RUMI	RUMM FINANCIAL BROKERAGE
29	SIJC	SPECIALIZED JORDANIAN INVESTMENT
30	SPIC	SPECIALIZED INVESTMENT COMPOUNDS
31	SURA	SURA DEVELOPMENT & INVESTMENT PLC
32	THBK	THE HOUSING BANK FOR TRADE AND FINANCE
33	THMA	TUHAMA FOR FINANCIAL INVESTMENTS
34	UBSI	BANK AL ETIHAD
35	UINV	UNION INVESTMENT CORPORATION
36	ULDC	UNION LAND DEVELOPMENT CORPORATION
37	UNAI	ARAB INVESTORS UNION CO. FOR REAL ESTATES DEVELOPING
38	UTOB	UNION TOBACCO & CIGARETTE INDUSTRIES
39	VFED	ALSHAMEKHA FOR REALESTATE AND FINANCIAL INVESTMENTS